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1994 J. Phys. A: Math. Gen. 27 L53

(http://iopscience.iop.org/0305-4470/27/2/009)

J. Phys. A: Math. Gen. 27 (1994) L53–L58. Printed in the UK

LETTER TO THE EDITOR

Edge states in a strong magnetic field

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Received 18 October 1993

Abstract. Energies and wavefunctions of edge states in a two-dimensional electron gas are evaluated for a finite step potential barrier model. The spectrum exhibits richer structure than observed previously for an infinite barrier model. Surprisingly, instead of smooth Landau level bending in the vicinity of the barrier, the levels acquire a step-like form. These plateaus have a direct impact on the widths of the magnetotransport conducting channels.

Strong magnetic fields allowed the discovery of such dramatic phenomena in a two-dimensional electron gas (2DEG) as the integer and fractional quantum Hall effects. In the microscopic theory [1] the edge states, classically represented by electrons skipping in circular segments along the edges, play a dominant role. Many magnetotransport experiments in a 2DEG have been qualitatively understood recently by means of a simple edge-state model [2]. This model is based on the picture of smooth Landau-level bending by the potential formed by external charges. The intersections of each Landau level with the Fermi surface create widely separated narrow edge channels [3]. Recently Chklovskii *et al* [4], using the self-consistent electrostatic approach, showed that the resulting effective potential should acquire a step-like shape. The Landau-level bending following these steps would transform the narrow edge channels into broader ones.

The semiclassical notion of the Landau-level bending is not always applicable to the electronic structure in the vicinity of the barrier representing an edge or an interface (or random potential). The existing explicit quantum mechanical model [5] is limited to the extreme case of an infinite barrier. This special barrier may fail to account for certain aspects of the surface/interface electronic structure, e.g. the existence of localized surface/interface states [6].

In this letter, a finite step potential is considered to better approximate the interface. This might be either be a 'boundary' confining a 2DEG or an interface between two different materials. The finiteness of the barrier confining the 2DEG is especially important if the confinement is realized by means of an interface between two similar materials. Then, in very strong magnetic fields the induced level splitting may reach comparable values.

Here, we study a system of 2D non-interacting electrons in the vicinity of a boundary under the homogeneous magnetic field B perpendicular to the xy plane. The edge or interface is described by a finite potential barrier between two materials (regions) L and R. In the region L the potential is lower by an amount V than in the region R. We assume that the interface is placed along the line $x=0$.

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0305-4470/94/020053+06$07.50

The one-particle Hamiltonian in the Landau gauge $A \equiv (0, Bx, 0)$ is

$$H = \frac{1}{2m}\left(p - \frac{e}{c}A\right)^2 + V\theta(x). \tag{1}$$

In this gauge the motion along the y direction is free and we can separate variables:

$$\psi_{n,X}(x, y) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{iXy}{a_L^2}\right) \phi_{n,X}(x) \tag{2}$$

where $\phi_{n,X}(x)$ obeys the one-dimensional equation

$$\left[-\frac{\hbar}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega_c^2(x-X)^2 + V\theta(x)\right]\phi_{n,X}(x) = E_{n,X}\phi_{n,X}(x) \tag{3}$$

where $a_L^2 \equiv \hbar c/eB$ is the magnetic length, $\omega_c \equiv eB/mc$ is the cyclotron frequency and X is the distance of the centre of a Larmor orbit from the interface. This is the Schrödinger equation for the harmonic oscillator superimposed with the step barrier. The integer n parametrizes discrete Landau levels. In the extreme case of an infinite step barrier, a simple boundary condition of the vanishing of the wavefunction is imposed:

$$\phi_{n,X}(0) = 0. \tag{4}$$

This was studied by McDonald and Středa [5], who obtained energies of first few levels as a function of the distance X. Deep inside the region L ($X \ll 0$) the influence of the interface is negligible and $E_{n,X} \to \hbar\omega_c(n+\frac{1}{2})$, n counting the Landau levels in the bulk. As X approaches the barrier, the energy levels rise due to repulsive effect of the infinite barrier. For orbits centred in the 'forbidden' region R the energies continue to rise indefinitely.

In systems of practical importance, the potential barrier is typically *not* much larger than the magnetic level spacing $\hbar\omega_c$. For example, in the 2DEG formed at the interface between GaAs and $GaAl_{1-x}As_x$, the effective interface potential barrier is only about 0.3 eV. The Landau level spacing in the strongest magnetic fields experimentally available is just a few times smaller.

When the interface barrier is finite, the simple boundary condition (4) should be replaced by a quantum mechanical matching of general solutions of (3) along the boundary line $x = 0$. Because of the explicit translational invariance in the y direction, the wavefunction matching for $\phi_{n,X}(x)$ has to be performed at a single point $x = 0$. It is convenient to shift the origin of the coordinate system to X; in natural units of magnetic length: $x' \equiv (\sqrt{2}/a_L)(x - X)$. The energy expressed in units of Landau spacing $\hbar\omega_c$ is $\varepsilon_n \equiv E_{n,X}/(\hbar\omega_c) \equiv \nu_n + \frac{1}{2}$. Equation (3) then takes a form

$$\left[\frac{d^2}{dx'^2} - \frac{1}{4}x'^2 - \frac{V}{\hbar\omega_c}\theta(x'+X) + \left(\nu_n + \frac{1}{2}\right)\right]\phi'_{n,X}(x') = 0 \tag{5}$$

which is the differential equation defining the parabolic cylinder functions [7]. The two linearly independent solutions $D_{\nu_n}(x')$ and $D_{\nu_n}(-x')$ satisfy asymptotically the conditions of rapid decrease for $x' \to +\infty$ and $x' \to -\infty$ respectively.

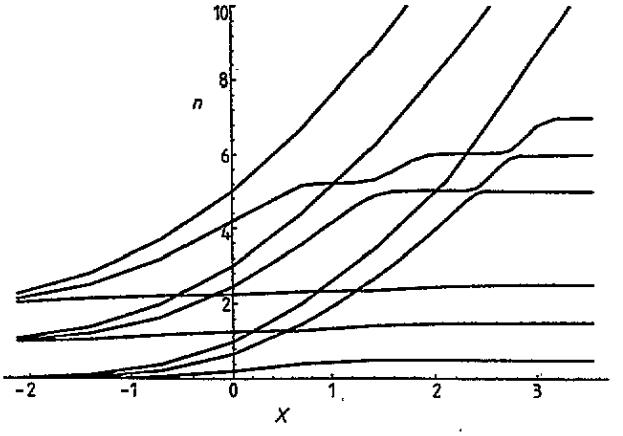

Figure 1. Eigenvalues of three lowest Landau levels, $E_x(X)=\hbar\omega_c(n(X)+\frac{1}{2})$. The energies are expressed in terms of $n(X)$, as functions of X, the centre of the Larmor orbit with respect to the barrier. Different curves correspond to three heights of the potential barrier: $V=0.5\hbar\omega_c$ (three at the bottom), $V=5\hbar\omega_c$ (three in the middle) and $V=\infty$ (three at the top).

The matching of the logarithmic derivatives at $x'=-X$ (or equivalently the condition of zero Wronskian in the expression for the Green function of the system) gives

$$\frac{D'_{\nu_n-\nu}(-x')|_{x'=-X}}{D_{\nu_n-\nu}(-X)}+\frac{D'_{\nu_n}(x')|_{x'=-X}}{D_{\nu_n}(-X)}=0. \tag{6}$$

This determines the energy levels ν_n as functions of the position X. The equation (6) was solved numerically using a simplified form†

$$D_{\nu_n+1}(-X)D_{\nu_n-\nu}(X)+D_{\nu_n-\nu+1}(X)D_{\nu_n}(-X)=0. \tag{7}$$

Eigenvalues ν_n for a few lowest Landau levels have been evaluated as functions of X, the centre of a Larmor orbit if the interface is not interfering, for two values of the interface potential V and are compared with those for an infinite wall [5]; see figure 1. Only the lowest three levels are shown. For small barrier $V=0.5\hbar\omega_c$ (three bottom curves) the levels simply bend a little. For an infinite barrier (three top curves) the levels continue to rise to infinity. The energies of states localized deep in the region L ($X\ll 0$) are not affected by the interface. They form the bulk Landau bands at $\nu_n=n$ and asymptotically do not depend on X. Qualitatively, when the radius of the classical Larmor orbit approaches the interface its repulsive effect pushes the energy upwards. Consequently at a given localization X, higher Landau levels start deviating from their bulk energies earlier than the lower ones due to their larger size $\simeq\sqrt{na_L}$.

Energies gradually saturate as the 'orbit centre' X moves deeper into the region R. Asymptotically, for $X\gg 0$ they reach values shifted by amount V with respect to those in the bulk of the region A. The transition over the interface is accomplished more slowly for higher Landau levels due again to their larger size.

The staircase shape of the curves for $V=5\hbar\omega_c$ (three middle curves in figure 1) can be understood as a tendency of each Landau level from region L to occupy the lowest

† Here well known relations expressing derivatives of the parabolic cylinder functions [7] were used.

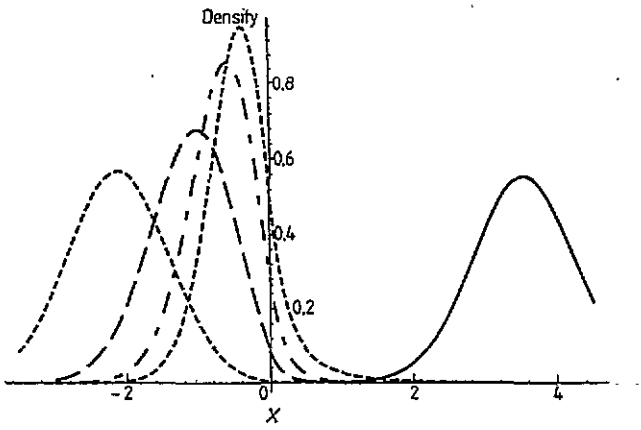

Figure 2. Electron density $|\phi_0(x)|^2$ for the lowest Landau level in the vicinity of the potential barrier. Larmor orbits are centred at five successive equidistant locations: $X=-2.12$ (short dashed), $X=-0.71$ (long dashed), $X=0.71$ (dot-dashed), $X=2.12$ (short dashed), and $X=3.54$ (solid curve).

level in R. The repulsion from the lower states pushes it towards the next plateau due to the non-crossing rule for electron levels. Note that the neighbouring plateaus are separated by very small gaps. We found numerically that the minimal gap between the lowest Landau level and first excited one is just $0.12\hbar\omega_c$.

Because of the nature of the mechanism just described†, the staircase type of transition from L to R is not restricted to just the finite-step potential. In fact, similar structures had been observed some time ago for a different potential. Heinonen and Taylor studied Hartree-type self-consistent solutions [8]. Their figure 2 indicates somewhat blurred features of plateaus.

Flat regions do not contribute to electric current and therefore spatially separated conducting edge channels are formed, similarly as in [4] although the mechanism is different from that in [4] and similar to that of Heinonen and Taylor [8].

In contrast to the infinite potential barrier when electrons are restricted to the region L only, here the wavefunctions penetrate into the region R. Far from the interface in the region R, when tunnelling is negligible, they restore their bulk shapes. Several typical probability densities of the states in the lowest and the first excited Landau levels in the vicinity of the step barrier are shown in figures 2 and 3 respectively.

The ground-state wavefunction, centred far from the barrier ($X=-2.12$, the leftmost curve in figure 2) is a Gaussian. As X approaches the barrier, the wavefunction becomes compressed. Two highest peaks, corresponding to orbits 'centred' on the barrier ($X=0.71$ and 2.12) clearly demonstrate how the centres of mass are lagging behind their classical Larmor orbit centres X. Only when the energy of a state reaches its saturation value inside B, is the Gaussian shape recovered ($X=3.54$, the rightmost curve in figure 2).

The next Landau level transformations in the region of the first energy plateau ($\nu\simeq 5\hbar\omega_c$, X around 2) are illustrated in figure 3. Two dashed curves ($X=1.77$ and $X=2.12$) are nearly Gaussians with only a small bump to the left from their node.

† The integer value of the barrier is not crucial to obtain the plateau structure: we observed similar structure for other values of V, too.

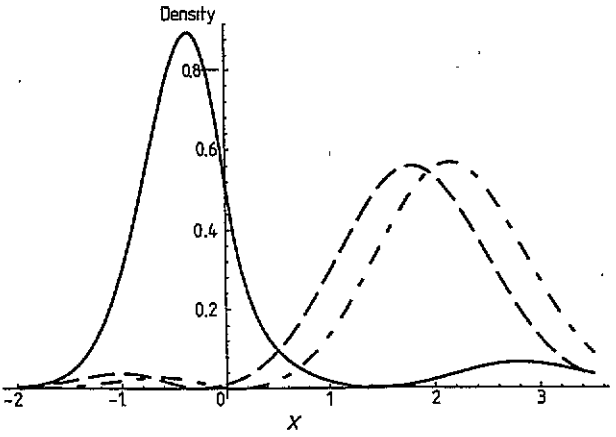

Figure 3. Electron density $|\phi_1(x)|^2$ for the first excited Landau level in the vicinity of the plateau of figure 1. Larmor orbits are centred at three successive equidistant locations: $X=1.77$ (dashed), $X=2.12$ (dot-dashed), and $X=2.48$ (solid curve).

When the orbit 'centre' is moved deeper into R ($X=2.48$, the solid line), after its energy is raised correspondingly, the electron centre of mass slips back into the region L left of the barrier. In this case, not only the centre of mass lags behind X, but even moves in the opposite direction.

The fact that the centre of the electron's wavefunction lags behind the orbital centre X when the latter is moved deeper into the region R is sometimes used as an argument for enhanced electric charge density at the edge [2]. However this lagging behind is not the only physical effect determining the charge density balance. Another relevant effect is the increased energy of the states which can become depopulated while crossing the Fermi level. Obviously, a proper self-consistent quantum mechanical calculation is needed to quantitatively determine the overall balance.

To summarize, energies of electron states approaching, in the strong perpendicular magnetic field, the finite-step potential barrier, representing an edge or interface in the 2DEG, have been investigated. In contrast to the infinite barrier model, the energies rise in steps (figure 1). The nth Landau level exhibits $n-1$ flat plateaus corresponding to $n-1$ lowest Landau levels in medium R. There is a curious 'apparent level crossing' in a sense that the higher level approaches very close to the lower ones. Flat regions do not contribute to electric current and therefore spatially separated conducting edge channels are formed, similarly as in [4] though the mechanism of their formation is completely different. Here we obtain the plateaus strictly as a one-electron effect without any corresponding structure of the potential.

We acknowledge the support of the National Science Council ROC, grants NSC-82-0208-M-001-116 (BR) and NSC-82-0501-I-00101-B11 (IB).

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